

Lecture 5: Search 4

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This Lecture

- ◆ Finish off Discussion of A*
- ◆ IDA*

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Admissibility and Monotonicity*

- ◆ Admissible heuristic $h(n)$ = never overestimates the actual cost $h^*(n)$ to reach a goal & $h(n) \geq 0$ & $h(\text{goal}) = 0$
- ◆ Monotone (Consistency) heuristic
 - For every node n and every successor n' reached from n by action a
 - $h(n) \leq \text{cost of } (n, a, n') + h(n')$ & $h(\text{goal}) = 0$
 - The deeper you go along a path the better (or as good) the estimate of the distance to the goal state
 - the f value (which is $g+h$) never decreases along any path.
 - Implies \implies Each state reached has the minimal $g(n)$
- ◆ When h is admissible, monotonicity can be maintained when combined with pathmax: $f(n') = \max(f(n), g(n') + h(n'))$
 - Create new $h \rightarrow h'(n') = \max(f(n) - g(n'), h(n'))$
 $f(n') = g(n') + h'(n')$
 - Force f to never decrease along path since $f(n') \geq f(n)$

Does monotonicity in f imply admissibility?

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Consistent $h \implies$ Monotone f

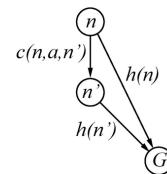
A heuristic is *consistent* if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) [= f(n)] \end{aligned}$$

i.e., $f(n)$ is nondecreasing along any path.



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Proof of Optimality of A*

Let O be an optimal solution with path cost f^* .
Let SO be a suboptimal goal state, that is $g(SO) > f^*$.
Suppose that A* terminates the search with SO .
Let n be a leaf node on the optimal path to O
 $f^* \geq f(n)$ **admissibility of h**
 $f(n) \geq f(SO)$ **n was not chosen for expansion**
 $f^* \geq f(n) \geq f(SO)$
 $f(SO) = g(SO)$ **SO is a goal, $h(SO) = 0$**
 $f^* \geq g(SO)$ **contradiction!**

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Completeness of A*

- ♦ A* is complete unless there are infinitely many nodes with $f(n) < f^*$
- ♦ A* is complete when:
 - there is a positive lower bound on the cost of operators.
 - the branching factor is finite.

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A* is maximally efficient

- ♦ For a given heuristic function, no optimal algorithm is guaranteed to do less work in terms of nodes expanded.
- ♦ Aside from ties in f , A* expands every node necessary for the proof that we've found the shortest path, and no other nodes.

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Questions*

- ♦ What are the implications of local monotonicity
 - Amount of storage
- ♦ What happens if $h_1 \leq h_2 \leq h$ for all states
 - h_2 dominates h_1
- ♦ If h_1 and h_2 are admissible, is $\max\{h_1, h_2\}$ admissible? Is it better than h_1 and h_2 ?
- ♦ What are the implications of overestimating h
 - Suppose you can bound overestimation
- ♦ What if you are doing a maximizing search

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Heuristic Function Performance

- While informed search can produce dramatic real (average-case) improvements in complexity, it typically does not eliminate the potential for exponential (worst-case) performance.
- The performance of heuristic functions can be compared using several metrics:
 - Average number of nodes expanded (N)
 - Penetrance ($P = d/N$)
 - Effective branching factor (b^*)
 - If solution depth is d then b^* is the branching factor that a uniform search tree would have to have to generate N nodes
 $(N = 1 + b^* + (b^*)^2 + \dots + (b^*)^d)$; **Given a d and N what is the b**
 - EBF tends to be relatively independent of the solution depth.
- Note that these definitions completely ignore the cost of applying the heuristic function.

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Measuring the heuristic payoff Iterative Deepening vs A*

d	Search Cost			Effective Branching Factor		
	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

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Meta-Level Reasoning

- ♦ Search cost involves both the cost to expand nodes and the cost to apply heuristic function.
- ♦ Typically, there is a *trade-off* between the cost and performance of a heuristic function.
 - E.g., we can always get a “perfect” heuristic function by having the function do a search to find the solution and then use that solution to compute $h(\text{node})$.

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Meta-Level Reasoning (cont'd)

- ♦ This trade-off is often referred to as the **meta-level** vs. **base-level** trade-off:
 - Base-level refers to the operator level, at which the problem will actually be solved;
 - Meta-level refers to the control level, at which we decide *how* to solve the problem.

We must evaluate the cost to execute the heuristic function relative to the cost of expanding nodes and the reduction in nodes expanded.

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IDA* - Iterative deepening A* (Space/time trade-off)

- ♦ A* requires open (& close?) list for remembering nodes
 - Can lead to very large storage requirements
- ♦ Exploit the idea the use of monotone f:
 - $\hat{f} = \hat{g} + \hat{h} \leq f^*$ (actual cost) and $f(n) \leq f(\text{next node after } n)$
 - *create incremental subspaces searched depth-first*
 - much less storage
- ♦ Key issue is how much extra computation
 - How bad an underestimate f , how many steps does it take to get $\hat{f} = f^*$
 - Worse case N computation for A*, versus N^2 for IDA*

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IDA* - Iterative deepening A*

- ♦ Beginning with an f-bound equal to the f-value of the initial state, perform a depth-first search bounded by the f-bound instead of a depth bound.
- ♦ Unless the goal is found, **increase the f-bound to the lowest f-value found in the previous search that exceeds the previous f-bound**, and restart the depth first search.
 - *Why if you reach a goal as a result of depth-first search is it optimal?*

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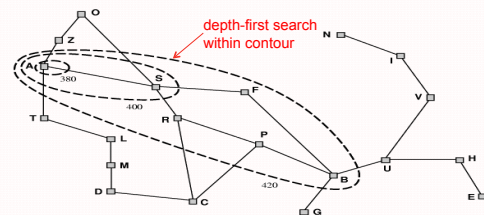
Iterative-Deepening-A*

- ♦ Algorithm: Iterative-Deepening-A*
 - 1) Set THRESHOLD = the heuristic evaluation of the start state.
 - 2) Conduct a depth-first search based on minimal cost from current node, pruning any branch when its total cost function ($g + h'$) exceeds THRESHOLD. If a solution path is found during the search, return it as the optimal solution.
 - 3) Otherwise, increment THRESHOLD by the minimum amount it was exceeded during the previous step, and then go to Step 2.
 - *Start state always on path, so initial estimate is always underestimate and never decreasing.*

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f-Cost Contours

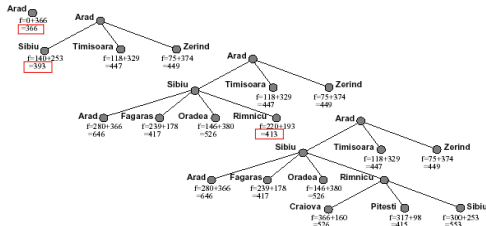
- ♦ Monotonic heuristics allow us to view A* in terms of exploring



- ♦ The more informed a heuristic, the more the contours will be “stretched” toward the goal (they will be more focused around the optimal path).???

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Stages in an IDA* Search for Bucharest



Nodes are labeled with $f = g + h$. The h values are the straight-line distances to Bucharest...

What is the next Contour???

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Experimental Results on IDA*

- IDA* is asymptotically same time as A* but only $O(d)$ in space - versus $O(b^d)$ for A*
 - Also avoids overhead of sorted queue of nodes
- IDA* is simpler to implement - no closed lists (limited open list).
- In Korf's 15-puzzle experiments IDA*: solved all problems, **ran faster even though it generated more nodes than A*???**
 - A*: solved no problems due to insufficient space; ran slower than IDA*

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RBFS - Recursive Best-First Search

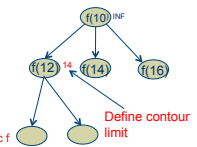
- Mimics best-first search with linear space
- Similar to recursive depth-first
 - Limits recursion by keeping track of the f -value of the best alternative path from any ancestor node - **one step look-ahead**
 - If current node exceeds this value, recursion unwinds back to the alternative path - **same idea as contour**
 - As recursion unwinds, replaces f -value of node with **best f -value** of children
 - Allows to remember whether to re-expand path at later time
- Exploits information gathered from previous searches about minimum f so as to focus further searches

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RBFS - Recursive Best-First Search Algorithm

function RECURSIVE-BEST-FIRST-SEARCH(*problem*) returns a solution, or failure
 RBFS(MAKE-NODE[INITIAL-STATE[*problem*]], ∞)

function RBFS(*problem*, *node*, *f-limit*) returns a solution, or failure and a new f -cost limit
 if GOAL-TEST[*problem*](*state*) then return *node*
 successors ← EXPAND(*node*, *problem*)
 if successors is empty, then return failure, ∞
 for each *s* in successors do $f[s] ← \max(g[s] + h[s], f[node])$; Pathmax heuristic; guarantee monotonic f
 repeat
 best ← the lowest f -value node in successors
 if $f[best] > f-limit$ then return failure, $f[best]$
 alternative ← the second-lowest f -value among successors Defines next highest f -contour
 result, $f[best]$ ← RBFS(*problem*, *best*, $\min(f-limit, alternative)$) Recursive search on best successor, remember when to backup
 if *result* ≠ failure then return *result*
 end



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Next lecture

- ◆ Other Time and Space Variations of A*
 - Finish off RBFS
 - SMA*
 - Anytime A*
 - RTA* (maybe if have time)